**Cover Sheet**

Title: *The Effect of Fiber Strength Stochastics and Local Fiber Volume Fraction on Multiscale Progressive Failure of Composites* for the *Proceedings of the* ***American Society for Composites—Twenty-Eighth Technical Conference***

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## ABSTRACT

Continuous fiber unidirectional polymer matrix composites (PMCs) can exhibit significant local variations in fiber volume fraction as a result of processing conditions that can lead to further local differences in material properties and failure behavior. In this work, the coupled effects of both local variations in fiber volume fraction and the empirically-based statistical distribution of fiber strengths on the predicted longitudinal modulus and local tensile strength of a unidirectional AS4 carbon fiber/ Hercules 3502 epoxy composite were investigated using the special purpose NASA Micromechanics Analysis Code with Generalized Method of Cells (MAC/GMC); local effective composite properties were obtained by homogenizing the material behavior over repeating units cells (RUCs). The predicted effective longitudinal modulus was relatively insensitive to small (~8%) variations in local fiber volume fraction. The composite tensile strength, however, was highly dependent on the local distribution in fiber strengths. The RUC-averaged constitutive response can be used to characterize lower length scale material behavior within a multiscale analysis framework that couples the NASA code FEAMAC and the ABAQUS finite element solver. Such an approach can be effectively used to analyze the progressive failure of PMC structures whose failure initiates at the RUC level. Consideration of the effect of local variations in constituent properties and morphologies on progressive failure of PMCs is a central aspect of the application of Integrated Computational Materials Engineering (ICME) principles for composite materials.

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## INTRODUCTION

While the use of polymer matrix composite (PMCs) in the aerospace industry has steadily increased in recent years, further development of Integrated Computational Materials Engineering (ICME) [1] approaches is needed to address some of the complexities associated with PMCs (*e.g.*, material property variability, processing/ fabrication variation, damage growth and evolution). Multifunctional PMCs have also been recently shown to exhibit unique characteristics that allow the simultaneous improvement of mechanical properties (*e.g.*, stiffness, strength, damage resistance) as well as functional properties (*e.g.*, thermal/ electrical conductivity, morphing) [2]. However, multiple sources of variation including fiber/ matrix properties and processing conditions contribute to the material uncertainty in PMCs. Such considerations are needed within a multiscale framework in order to develop ICME-based approaches which can be used to predict and assess the damage tolerance capabilities of multifunctional composites [3].

The Micromechanics Analysis Code with Generalized Method of Cells (MAC/GMC) [4] provides a computationally efficient means of modeling composite materials based on Aboudi's method of cells [5-8]. Using the GMC, a doubly or triply periodic repeating unit cell (RUC) is discretized into an arbitrary number of subcells. Each subcell is assigned material properties and a constitutive law to describe the local material behavior. Continuity of displacements and tractions are then enforced along the subcell boundaries in an average sense, and all field quantities are evaluated at the subcell centroids. For example, Fig. 1 contains the microscale representation of a unidirectional composite comprised of continuous fiber(s) embedded in a homogenous matrix. The actual microscale of the PMC consists of a square-packing arrangement of fibers as shown in Fig. 1a. This microsctructure can then be represented in MAC/GMC by discretizing the domain into a series of fiber and matrix subcells. Figures 1b and 1c show the microscale representation of a single-fiber and four-fiber doubly periodic RUC, respectively. Note that no stress concentrations are introduced at the corners of the rectangular subcells since field quantities are evaluated at each subcell centroid rather than the subcell corner points. MAC/GMC may also be coupled to ABAQUS Standard or Explicit [9] by another code, FEAMAC [10, 11]. Using this coupling technique, finite element integration point strains are mapped onto RUCs and a local MAC/GMC analysis is performed. Subsequent changes in material response are then passed back up to the finite element level and the procedure continues. This technique is graphically shown in Fig. 2.

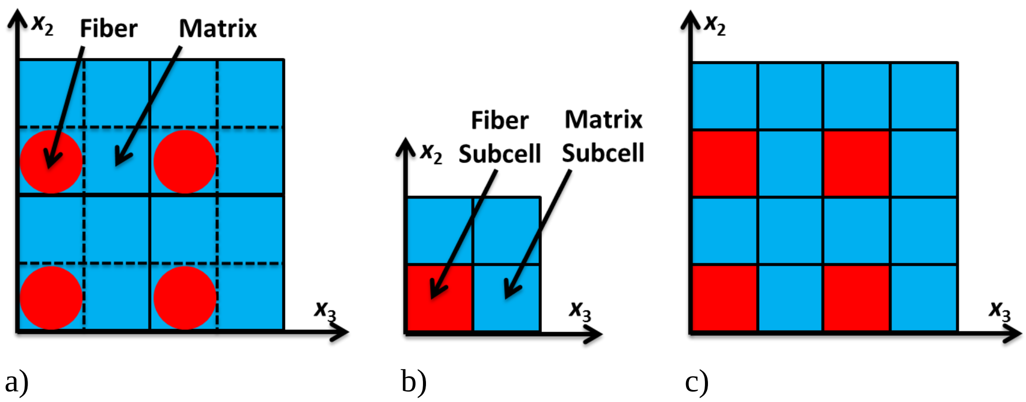


**Figure 1.** a) Square-packing representation of a unidirectional PMC. Microstructural representation using b) a single-fiber RUC and c) a four-fiber RUC.

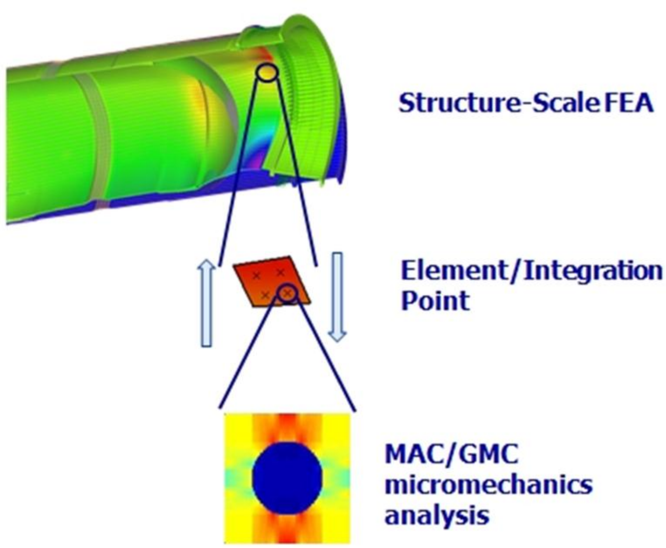


**Figure 2.** Schematic showing the coupling of MAC/GMC with ABAQUS via FEAMAC.

Recently Ricks *et al*. [12, 13] developed a multiscale modeling methodology using MAC/GMC and FEAMAC for predicting the ultimate strength and failure behavior of composite materials, where a local distribution of fiber strengths was incorporated at the RUC level prior to performing a global progressive failure analysis at the specimen coupon level. Individual fiber strengths were determined from a modified two-parameter Weibull cumulative distribution function (CDF) developed by Watson and Smith [14] and Padgett *et al*. [15] in order to characterize the fiber strength distribution across a range of fiber segment lengths. This modified Weibull CDF shifts the distribution towards higher strength values (*i.e.*, decreased probability of a flaw along the fiber segment) as the fiber segment length decreases. Ricks *et al*. [12] found that for a SCS-6/ TIMETAL 21S metal matrix composite tensile dogbone specimen, for a constant finite element mesh density, increasing the RUC complexity (*i.e.*, more fiber subcells at a constant fiber volume fraction) resulted in an increase in the macroscale ultimate tensile strength (UTS) and more randomly distributed fiber failures throughout the specimen. A similar trend was observed for the PMC coupon specimens considered in Ricks *et al*. [13]. However,

additional sources of material uncertainty can contribute to the global composite response that should be addressed within a global ICME framework. As a result of processing conditions, PMCs typically exhibit local fluctuations in fiber volume fraction as a result of randomly distributed fibers. These local differences in fiber volume fraction can lead to further differences in local material properties as well as failure behavior. In this study, the coupled influence of the local variation in fiber volume fraction and fiber strengths on the predicted RUC-averaged strength of a unidirectional carbon-epoxy composite is investigated using MAC/GMC for an AS4 carbon fiber/ Hercules 3502 (AS4/ 3502) epoxy compsoite. Future work will investigate the coupled effects of these two sources of variation on the macroscale (*i.e.*, global) composite response in multiscale progressive failure analyses as well as to generalize the methodology to account for additional sources of variation.

## MATERIAL SYSTEM

For this study, a unidirectional AS4 carbon fiber/ Hercules 3502 (AS4/ 3502) epoxy material system was considered. Both the fiber and matrix were considered isotropic and linear elastic materials. The fiber has a Young's modulus of 234 *GPa* and Poisson's ratio of 0.2 while the matrix has Young's modulus of 3.8 *GPa* and Poisson's ratio of 0.36 consistent with experimental data [16, 17].

Two sources of material variation were considered in this study: fiber tensile strength and local fiber volume fraction. Future studies will include additional sources of uncertainty due to local fiber and matrix moduli, matrix strengths, and other factors. In order to characterize the fiber strength, a statistical distribution of fiber strengths was assigned to individual fiber subcells within a given RUC consistent with the methodology developed in Ricks *et al*. [12, 13]. These strengths were determined by solving the following modified two-parameter Weibull cumulative distribution function [14, 15] for the fiber strength, σ,

$$P_f(\sigma) = 1 - exp\left[-\left(\frac{L}{L_0}\right)^{\alpha}\left(\frac{\sigma}{\sigma_0}\right)^{\beta}\right] \quad (1)$$

where $P_f$ is the cumulative probability of failure at an applied tensile stress, $\sigma$, and is taken to be a random number (*i.e.*, [0,1]). The traditional two-parameter Weibull scale ($\sigma_0 = 4493$ *MPa*) and shape ($\beta = 4.8$) parameters were determined from experimental data based upon a fiber test length ($L_0 = 10$ *mm*) [18]. The fiber strength parameter, $\alpha = 0.6$, is uniquely determined from experimental data where multiple fiber lengths were tested [18]. $L/L_0$ represents the ratio of some characteristic length, $L$, of interest to the tested fiber length, $L_0$. In order to ensure the fiber segment lengths simulated at the microscale appropriately represented the continuous fibers existing at the macroscale in multiscale analyses, $L$ was selected to be equal to the finite element length; see Ricks *et al*. [12, 13] for additional discussion of this issue. In the current work, however, since no multiscale simulations were performed, the classic two-parameter Weibull CDF (*i.e.*, $L/L_0 = 1$ in Eq. 1) was used to assign fiber strengths to individual subcells within an RUC. A maximum stress failure criterion was used to determine fiber failure. No matrix

failure criterion was implemented in the current study; this will be addressed in future work.

A mean fiber volume fraction of 0.6 was used in these analyses consistent with data from the literature [19]. It was assumed the local fiber volume fraction could be represented by a normal (*i.e.*, Gaussian) distribution with an 8% coefficient of variation (CV) based upon work performed by Chamis [20]. The local distribution in fiber volume fraction could also be characterized by using high resolution images of the composite if available. Additionally, the amount of variability in the measured local fiber volume fraction is dependent on the averaging window used. Hence, the variability would decrease as the size of the averaging window increased (*i.e.*, a more uniform fiber volume fraction). Such considerations will be addressed in future work.

## RESULTS

Multiple doubly periodic RUCs (*cf*. Fig. 1) were considered as part of this study: single-fiber, four-fiber, nine-fiber, 16-fiber, and 25-fiber RUCs. The later RUC was previously shown to provide a converged estimate of the local strength for this material system [13]. Three sets of RUC-level (*i.e.*, local) calculations were performed. First, the fiber strength was allowed to vary while maintaining a constant 60% fiber volume fraction. Second, the local fiber volume fraction was allowed to vary while maintaining a constant fiber strength of 4116 *MPa*. This strength value is equal to the mean of the Weibull distribution represented by Eq. 1. For these two cases, 500 simulations were performed for each RUC. Lastly, both the fiber strength and fiber volume fraction were allowed to simultaneously vary and 2500 simulations were performed for each RUC. The predicted longitudinal modulus and local strengths of the RUC were determined from the results. Since a change in the fiber strength alone does not produce an initial change in the longitudinal modulus (*i.e.*, prior to the onset of fiber failure) in the composite, the longitudinal modulus was only determined for the two cases where the fiber volume fraction varied. Additionally, a random sampling methodology was employed for all three cases. As a result, a large number of simulations are needed to sufficiently encompass the possible combinations of random variables (*i.e.*, fiber volume fraction/ strength). Future work will investigate the use of a more complex sampling methodology (*e.g.*, Latin hypercube sampling) that typically generates reliable results in fewer simulations than random sampling [21]. This sampling issue becomes particularly important when additional sources of variation are considered.

Figure 3 contains a plot if the mean longitudinal modulus of the RUC as a function of the number of fibers. Error bars are also shown and denote the minimum and maximum longitudinal modulus values obtained. Note that as the number of fibers increased, no significant differences in the mean longitudinal modulus were observed. The variation in longitudinal modulus, however, decreased with increasing numbers of fibers; this is due to the decreased influence of any one fiber on the RUC-averaged modulus. Additionally, for the case where both the local

volume fraction and strength varied, the mean longitudinal modulus matched that for the case where only the local volume fraction varied. This makes sense since the only source of variation in the initial homogenized stiffness can be attributed to the fiber volume fraction. While the mean RUC-averaged longitudinal modulus did not significantly vary in these simulations, substantial variations may be observed if the coefficient of variation in the local fiber volume fraction is increased and additional sources of elastic moduli variation are permitted (*e.g.*, variations in fiber/ matrix moduli, matrix strengths, fiber-matrix interfacial properties, etc.). Figure 4 contains a plot of the mean RUC-averaged strength as a function of the number of fibers. When only the fiber strength was varied (*i.e.*, the fiber volume fraction was held fixed), the mean local composite strength decreased with increasing numbers of fibers. Similarly, the variation in the RUC-averaged strength decreased as more fibers were added to a RUC. Since the influence of any one fiber on the RUC-averaged strength decreases as the number of fibers increases, both the magnitude and variation in the mean local composite strength decreased as more fibers were simulated in a given RUC. These values would asymptotically approach the macroscale continuum-averaged response once the number of simulated fibers became appropriately large, consistent with the results reported by Ricks *et al*. [12, 13]. If only the fiber volume fraction varied, however, no significant changes in the mean local composite strength were observed, and the variability in strength decreased slightly as the number of fibers increased. When both the fiber strength and fiber volume fraction were allowed to simultaneously vary, the RUC-averaged composite strengths nearly match the case where only the fiber strength was varied. Although the variability in strength increased slightly for the case where both the fiber strength and volume fraction varied simultaneously, this can primarily be attributed to a larger sampling size (*i.e.*, 2500 simulations versus 500 simulations for the case where only fiber strength was varied). Additionally, if a larger distribution of local fiber volume fractions were considered (*e.g.*, 20-40% coefficient of variation), more significant differences in the predicted strengths would likely be observed. A key concern is to account for local fluctuations in constituent properties, morphologies, and volume fractions when performing composite multiscale progressive failure analyses within an ICME framework. This topic is the focus of ongoing work. Such considerations will be crucial to the further development of ICME-based approaches for composite materials.

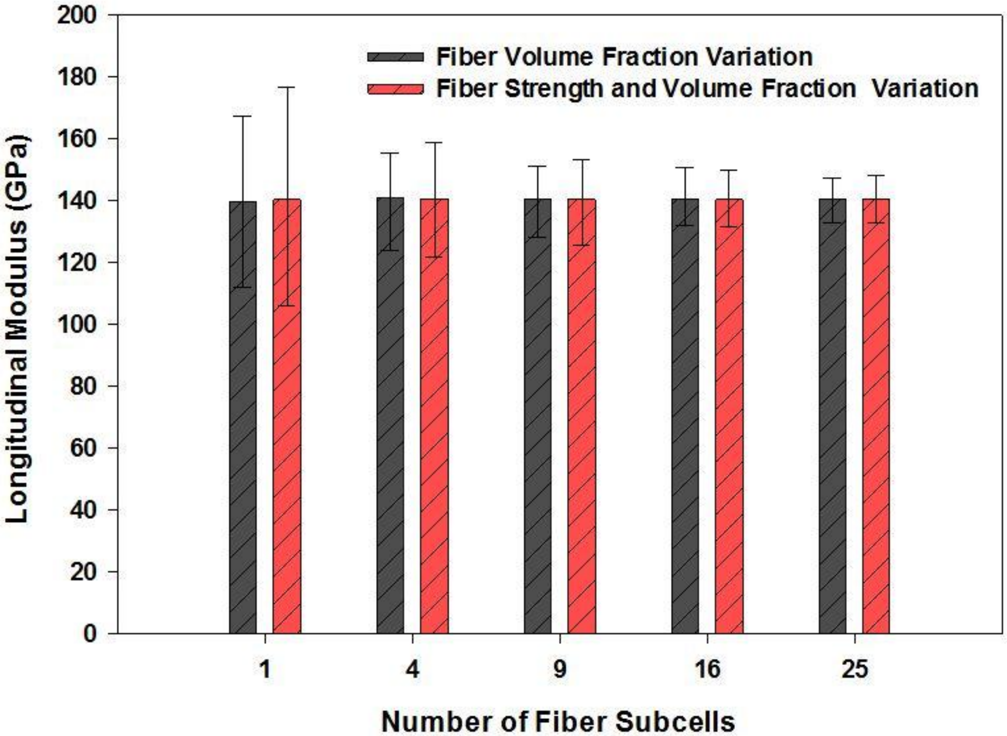


**Figure 3.** Mean longitudinal modulus as the number of fiber subcells increases for the cases where only the fiber strength, only the fiber volume fraction, and both were simultaneously varied.

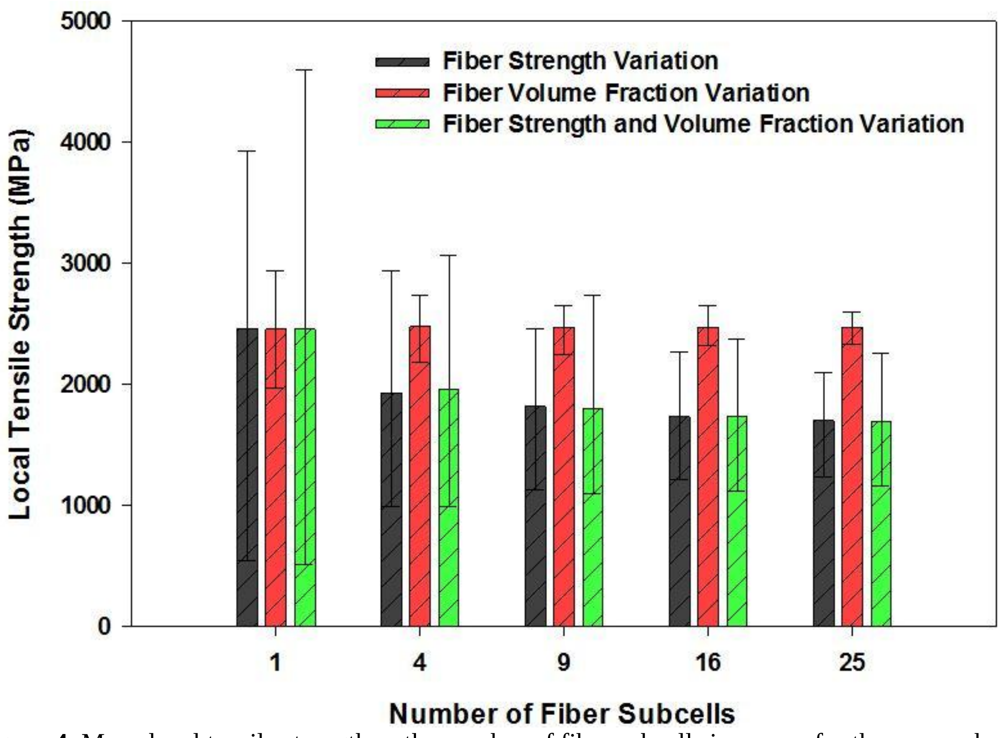


**Figure 4.** Mean local tensile strength as the number of fiber subcells increases for the cases where only the fiber strength, only the fiber volume fraction, and both were simultaneously varied.

## SUMMARY AND CONCLUSIONS

In this study, the effects of local variations in fiber tensile strength and volume fraction on the effective composite properties for a unidirectional AS4 carbon fiber/ Hercules 3502 epoxy composite were investigated. Predicted local longitudinal moduli and tensile strengths were determined by homogenizing the composite material response over repeating unit cells (RUCs) containing increasing numbers of simulated fibers. Small changes (~8%) in local fiber volume fractions did not significantly affect either the RUC-averaged mean composite strength or longitudinal modulus. When both the local fiber volume fraction and fiber strength were allowed to simultaneously vary, the local RUC-averaged strength was strongly influenced by the local distribution of fiber strengths; the composite strengths were relatively insensitive small changes in local fiber volume fraction. Similarly, since changes in the fiber strength do not contribute to the initial composite elastic properties, the predicted longitudinal moduli were not appreciably affected by changes in local fiber strengths. The RUC-averaged constitutive response, which accounts for local variations in constituent properties and morphologies, can be used to characterize lower length scale material behavior within a multiscale analysis framework; this will be the focus of future work. By simulating a more realistic representation of as-fabricated polymer matrix composites, a crucial step is made in the further development of Integrated Computational Materials Engineering (ICME) approaches for composite materials.

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